NONLINEAR STUDIES - www.nonlinearstudies.com
Vol. 26, No. 4, pp. 707-722, 2019

I&S PUBLISHERS

# Lars-Erik Persson - the remarkably broad and innovative mathematician and unique Pers(s)on

Dag Lukkassen [1*], Annette Meidell [1]

[1] Department of Computer Science and Computational Engineering
UiT The Artic University of Norway, Campus Narvik, Norway
E-mails: Dag.Lukkassen@uit.no, Annette.Meidell@uit.no

[*] *Corresponding Author.* E-mail: Dag.Lukkassen@uit.no

**Dedicated to the 75th anniversary of Professor Lars-Erik Persson**

**Abstract.** We laconically describe the contribution of Lars-Erik Persson to mathematical research and postgraduate studies. In particular, we describe and give examples of his innovative and great contributions to the following areas: Fourier analysis (or more generally harmonic analysis), interpolation theory, homogenization theory, Hardy type inequalities, convexity and inequalities, and applied mathematics together with a brief description of his historical and didactic related contributions. Moreover, we will describe and give concrete examples of his unique way to work together with PhD students both before and sometimes also after their exams. Finally, we will shortly describe his remarkable "class travel" from a small village in north Sweden to finally become the broad, innovative and powerful mathematician he obviously is regarded to be today. We also refer to the article [A] about Lars-Erik as a great person and great mathematician, which was published in a journal issue dedicated to his 65th anniversary. We essentially update, complement and broaden the information given in [A] especially in the part concerning Lars-Erik Persson's contributions to mathematical research and PhD education.

## 1 Introduction

First of all we want to pronounce that it is in this limited amount of pages absolutely impossible to give a fair description of Lars-Erik Persson (LEP) concerning his work, impact in the mathematical

[2010] **Mathematics Subject Classification**: 01A99, 46E30, 46B70, 42A16, 42B35, 43A50, 26A51, 26D10, 26D15, 46E99, 54C35, 97M10

**Keywords**: Lars-Erik Persson, Fourier analysis, harmonic analysis, Interpolation theory, inequalities, Hardy-type inequalities, convexity, function spaces, functional analysis, applied mathematics, history and didactical related research, PhD education.

community and his really unique and strong character. But to get a first glimpse of this fact we recommend the reader to visit his new international homepage **http://www.larserikpersson.se**.

In June 8-11, 2009, there was organized the conference "Analysis, Inequalities and Homogenization Theory (AIHT) - midnightsun conference" in Luleå, Sweden, in honour of LEP on the occasion of his 65th birthday. In the article [A] published in the journal issue dedicated to this event there was described some of LEP's contributions in interpolation theory, functional analysis and Hardy-type Inequalities with special focus on the collaboration with the author. It was also mentioned briefly some contributions within the area homogenization theory. We essentially update and complement the information given in [A] concerning LEP's contributions to the above mentioned areas. But the main aim is to broaden the picture of LEP in another direction namely we will shortly describe and exemplify LEP's corresponding great and innovative contributions also to the areas Fourier analysis (or more generally harmonic analysis), convexity (with special focus on the relation to inequalities) and applied mathematics, together with a brief description of his historical and didactic related contributions both in practise and in publications. One other aim is to further pronounce LEP's especially important contributions in the mission to encourage young talented young students to devote their lives to mathematics. He has so far been a supervisor for 63 students with PhD exams out of which 10 nowadays are professors (LEP is still very active and at the moment a supervisor for six PhD students so these numbers will for sure increase). For several of these students LEP has not only been a supervisor but also a mentor and main collaborator also after their theses defences, in some cases even closely related to their own careers to be professors. The authors of this article are two such examples and our own experiences are shortly described in Sections 6. In July 15-18, 2014, a special Workshop dedicated to 70th anniversary of LEP was organized in the frameworks of the world congress ICNPAA 2014, which was held at UiT, The Arctic University of Norway, Campus Narvik. Professor Pierre-Louis Lions gave the opening plenary lecture dedicated to LEP. This was the starting point of an important new scientific contact and friendship for LEP. For example, in November 2015 LEP was invited to deliver a series of lectures in the special seminar of Professor Lions (where at most one researcher from abroad is invited in this way each year). So, indeed LEP is in the unique situation to be closely related to both generations of famous Professors Lions.

**fig.1:** In the photo you see Lars-Erik Persson together with his friend and important scientific contact Professor Pierre-Louis Lions in his office in Paris in connection to the above mentioned invitation to his seminar (c.f. also [31]).

In July 22-26, 2019, special Session dedicated to 75th anniversary of LEP was organized in the frameworks of the international conference IWOTA2019, which was held in Lisbon, Portugal. Moreover, at this conference LEP was invited plenary speaker and delivered his lecture with the title "My life with Hardy and his inequalities".


**fig.2:** This photo is from the banquet at IWOTA2019. In particular, in front of LEP are Professors Ryskyl Oinarov and Vladimir Stepanov, which are especially important collaborators of LEP since they and LEP have jointly supervised around 15 students to their PhD exams. To the left you see another important collaborator of LEP, Professor Natasha Samko, one of the organizers of the Session dedicated to LEP at IWOTA 2019.

After these introductory words we are ready to present the remaining parts of our updating and complements of the text in [A], which we hope gives a more complete and broad picture of the really unique, generous, innovative and happy mathematician Lars-Erik Persson. The paper is organized as follows: In Section 2 we briefly describe the unique "class travel" of LEP from his small home village (Svanabyn) to be a famous professor in mathematics. As a support of this, in Section 3 we present some facts from LEP's strong CV. In Section 4 we describe LEP's really unique merits concerning PhD education. As a further support of this, in Sections 5 we describe our own experiences of how LEP works and behaves as supervisor, research collaborator and supporting friend. The most important part of this paper is Section 6, where we shortly describe and exemplify LEP's most important merits in research within the following areas; Fourier analysis, interpolation theory, homogenization theory, Hardy type inequalities, convexity and inequalities, functional analysis and function spaces and applied mathematics together with some of his historical and didactic related contributions. Finally, Section 7 is reserved for some final words about LEP, especially in honour to his 75th anniversary, and in Section 8 we list the selection of his papers and books we have referred to in the text.

## 2 Brief description of the life and unique "class travel" of LEP

Some of the information in this Section can also be found in [A] but we have also included some important complements and new information. LEP was born on 24 September 1944 in Dorotea but the family was living in Svanabyn, which is a village 30 km from Dorotea with around 150 inhabitants. He finished here classes 1-6 and he was the first person from Svanabyn who went for further studies in Dorotea. In fact, during his time in Svanabyn LEP had never heard about the fact that there existed something called "University".

LEP lived under especially hard conditions these first years. His father worked in the forest and was not living at home long periods and in this village there were no works with income for females. Moreover, LEP could have had eight brothers and sisters but finally only LEP survived. One problem concerning this remarkable fact was these really hard conditions but another one was also a at that time not known problem, the so called "blood conflict". But his really fantastic mother Dagmar never gave in e.g. in her fight to keep at least LEP. We have never seen LEP without tears in his eyes when he talk about his fantastic mother and it is clear that this has essentially influenced the development of his unique character.

Fortunately, the doctor in Dorotea Sten Strmbom miraculously enough finally understood this conflict and could help Dagmar so she could keep LEP. But Dr. Strömbom was also extremely important person for LEP for other reasons. The first step was that LEP could stay in his home in Dorotea almost as adopted son during his studies at "realskolan" in Dorotea (corresponding to classes 7-9). Dr. Strömbom understood quickly LEP's unique talent and potential and from him LEP picked up and learnt a lot of crucial things, which has influenced LEP in all his life. In particular, LEP understood that it could be a door open even for him to enter the academic world. It is also clear that also this period was extremely important to create a bases for LEP's unique character.

After finishing his studies in Dorotea with highest marks LEP continued his studies 1961-1964 at the gymnasium in Östersund in the middle of Sweden. After that, in 1964, he did his military service in Umeå. The next five years (1965-1969) he studied mathematics, physics and other related subjects at Umeå University. But already at first year studies in mathematics LEP's talent in this subject was observed and he even got the offer to be part-time "amanuensis" (help teacher) at the department of mathematics. In 1970 he finished his teacher education studies at Umeå University and was even working half a year at a secondary school as a part of this education. But already during these teacher studies LEP was asked to be PhD student in mathematics. First LEP thought they joked, the modest boy from small Svanabyn could hardly get such fin title as doctorand. But of course LEP accepted the offer. His supervisor was professor Ingemar Wik, a former student of the famous Swedish professor Lennart Carleson. So, in a way, LEP is scientific grandson of professor Carleson. The PhD studies of LEP worked very well and as can be seen from the descriptions in Section 6 he already then initiated and developed many ideas, which was of importance also after that he finalized his PhD studies in 1974 with the PhD thesis [1].

In the academic year 1974/75 LEP held a temporary position as associate professor and was teaching at Umeå University and from 1975 he became permanent associate professor at Luleå University of Technology. In 1987 he became docent (corresponding to habilitation in some other countries). In 1988-1992 LEP worked as acting professor at Luleå University of Technology. Moreover, in 1992 he was appointed as full professor at Narvik University, Norway (now a part of UiT The Artic University of Norway). He held this position till 1994 and from then and still he has worked as part-time professor at Narvik University/UiT The Arctic University of Norway. In 1994 LEP was appointed as full professor at Luleå University of Technology and worked on this position till 2011. In the period

2011-2018 he worked as senior professor at the same university. Moreover, in the period 2004-2010 he worked also as part-time professor at Uppsala University, Sweden. In 2019 he was appointed as senior professor at Karlstad university, Sweden. Finally, it should be mentioned that in 2005 he was appointed as honorary professor at Eurasian National University in Astana, which is the capital in Kazakhstan.

During the remarkable academic travel described above LEP has got merits presented in a correspondingly very strong CV, see section 3. Hence, it is natural to call this a unique "class travel" from LEP's "poor" conditions in the small village Svanabyn till now have been established as a strong first class mathematician.

## 3 Some information from LEP's CV

* LEP was vice President (1994-1996) and then President (1996-1998) of the Swedish Mathematical Society.
*In the period 1995-2002 LEP was been the secretary of the Swedish National Committee for Mathematics at the Royal Academy of Science.
* LEP was ordinary member six years of the board NT1 (Mathematics and technical mathematics) at The Swedish Research Council, which is a government's agency that distributes the funding for basic research of the highest quality in Sweden. Four of these years (2010, 2012-2014) he was even the chairman of the board.
* LEP has been a supervisor of 63 students with PhD exams (see Section 4 for more detailed information). Moreover, in addition he has been a supervisor of 12 students with Licentiate exams (this is a Swedish exam two years after Master level and thus something like half PhD). At the moment he is a supervisor of 6 PhD students.
*LEP has cooperated with several researchers around the world. In particular, he has written joint papers/books with around 170 mathematicians from 39 different countries.
*LEP is the author or co-author of around 350 scientific publications, out of which 16 are books. Moreover, he has been an Editor of 7 books.
*LEP is editor for six international Journals. One of them is "Journal of Function Spaces and Applications". In fact, he was the founder and Editor-in-Chief for this Journal up to 2010.
*LEP has been invited one month or more to around 10 universities. The most prestigious invitation is that when LEP was invited in November 2015 to deliver a series of lectures in Pierre Louis Lions' seminar (see [31]). At most one researcher from abroad is invited each year.
* LEP has been opponent or in grading committee for around 30 PhD examinations.
*LEP was the founder of "Centre of Applied Mathematics" at Luleå university of technology and he was its director up to 2005
*LEP was director of "Centre of Interdisciplinary Mathematics" at Uppsala university during 2007-2009.
*LEP has been a member or head of the organizing committee for five international conferences. Moreover, he has delivered around 50 talks at international conferences, where around 20 of them are given as plenary or specially invited talks.
*LEP has been external expert to the appointment boards for around 50 academic positions at 14 different universities. Six of these positions are full professorships.
*LEP has been opponent or in grading committee for around 30 PhD examinations.
* LEP obtained in 2002 "Louise Petrén's award" for his special support to female mathematicians in Sweden (In particular, he has been a supervisor for 29 females with PhD exams). Remark: LEP was

the first one who got this award.
*LEP obtained the Orlicz medal in 2003 for his contributions in mathematical research.
*LEP received Sammy Lindmark's award 2004 (50 000 SEK) for his research and cooperation with the society outside the university.
*LEP was appointed as honorary Professor in 2005 at Eurasian national University in Astana, Kazakhstan.
* LEP received the Luleå university of technology award 2005 (40 000 SEK) for his outstanding cooperation with the world outside outside the university.
* LEP received the Luleå university of technology award 2008 (10 000 SEK) as the best supervisor of the university. Remark: LEP was the first one who got this award.
*LEP received "Ångpanneföreningens" award 2008 (100 000 SEK) for his outstanding work to transfer important knowledge to the world outside university. Remark: This national award is given each year to at most one researcher from Sweden in a competition between all subjects. This is the only time it has been given to a mathematician.

Finally, we want to mention that the work of LEP has been recognized in various ways also outside the university community. In particular, it has been several articles about him in Swedish newspapers and he has appeared in Swedish radio and television many times. For example in one of these television programs, he was appointed as expert in a debate e.g. with the two representatives from the Swedish government, which decides over all money for education in Sweden. In fact, it is clear that LEP is one of the most known mathematicians also outside the university community in Sweden, maybe even the most known.

## 4 LEP and his unique contributions as supervisor

The following 63 students have finalized their PhD exams with LEP as a supervisor:

1992 Nils Svanstedt**; 1994 Thomas Strömberg*; 1996 Andrejs Dunkels, Anders Holmbom**, Dag Lukkassen**; 1997 Stefan Ericsson; 1998 Niklas Wellander*, Marianna Euler*, Peter Wall**; 2000 Sorina Barza**; 2001 Ove Lindblom, Annette Meidell**; 2002 Johan Bystrm, Maria Nassyrova; 2003 Niklas Grip*, Leon Simula, Dmitry Prokhorov, Leo Larsson; 2004 Anna Wedestig; 2005 Evgueny Kuznetsov, Jonas Engstrm, Narsis Mtega; 2006 Aigerim Kalybay*, Christopher Okpoti, Elena Ushakova, Monica Johansson, Kristina Juter**, Orjan Hansson; 2007 Komil Kuliev, Gulchehra Kulieva, Alice Lesser; 2008 Maria Johansson*, Emmanuel Kwame Essel; 2009 Zamira Abdikalikova, Guy Beeri, Irina Pankratova, Tomas Johnson; 2010 Yulia Koroleva, Per-Eskil Persson, Anca-Nicoleta Marcoci, Liviu-Gabriel Marcoci, Kristina Krulic, Klas Pettersson; 2011 Lyazzat Sarybekova. John Fabricius; 2012 Olga Popova, Alessandra Ragusa; 2013 Larissa Arendarenko; 2014 Olov Viirman; 2015 Ainur Temirkanova, George Tephnadze; 2016 Mervis Kikonko, Akbota Abylayeva, Afonso Fernando Tsandsana. Martin Krepela; Mira Randahl, Nina Lintz 2017 Ralph Höibakk, Aigerim Kopezhanova 2018 Raya Akhmetkaliyeva, Guldarya Shambilova, Serikbol Shaimardan, Staffan Lundberg.

The students marked with ** have so far become full professors and those marked with * "Biträdande Professor" (corresponding to Professor internationally). LEP is still very active as supervisor, at the moment he is a supervisor for six PhD students at three different universities.

The starting point of this adventure was around 1988 when LEP got financial support and recruited his first PhD student Nils Svanstedt (later on Professor at Chalmers university, Gothenburg, Sweden)

and they worked closely together during the period LEP learnt and introduced homogenization theory as an independent subject in Sweden and Norway.

LEP works many times in close collaboration with an international net of researchers. For around half of the students mentioned above the supervision has been performed jointly with some supervisor from abroad. Totally around 10 professors from abroad have jointly supervised students with LEP. Among them we mention especially professors Vladimir Stepanov (Russia) and Ryskul Oinarov (Kazakhstan) since they have jointly with LEP supervised totally around 15 students to their PhD exams. The students above come from 10 different countries: Check Republic, Norway, Ghana, Kazakhstan, Mozambique, Romania, Russia, Sweden, Tanzania and Uzbekistan.

LEP got the prestigious award as the best supervisor (all subjects) at Luleåuniversity of technology first time this award was established in 2008.

## 5 Our own experience of LEP

Sometimes LEP involves himself not only in the scientific work of his PhD students, but also in other parts of their lives. For example, several foreign PhD students, and sometimes also colleagues, have been invited to stay in his house for shorter or longer periods. Two PhD students from Africa even stayed there for three years during all their PhD studies. We asked LEP: How can you afford to feed all these people? LEP answered: We eat the food we have. For example, I pick more than 100 kg berries each year and get a lot of fish too. Sometimes my PhD students join me on these trips. So we know that there is always food on the table, and an especially friendly, generous and creative atmosphere in this house. We also know that LEP is proud that his house therefore got the name "Hotel Infinity". LEP considers you differently from what you may be used to. He never puts an interest in your weaknesses. That is a waste of time, he says. What he seeks for is your strong sides, those that make you unique. He also understands that a person's career does not only depend on academic skills, but also on the way we deal with family situation, our feelings and the care that we share with each other.

LEP has always known that the best way to obtain good results and achieving his aims is by helping others to achieve theirs. -Walk side by side with your colleagues but remember that you yourself reach higher if you can motivate and help them to reach their own goals.

LEP is a survivor. He has a general rule. Even if your position is poor, you must never be tempted to give up. Instead, you should always seek for the optimal state of any situation. If you are together with the most famous mathematicians, use the opportunity for your own (and theirs) benefit. If you get problems on your present job, or have not sufficient with money, just look for other creative solutions. There is always a better solution available in each given situation than the one you can picture in your darkest fears.

Both authors of this article have known LEP and his unique character for a very long time starting with the period with him as a very supporting and effective supervisor. Our own experience of LEP is that he has been an important collaborator and supporting friend to us not only in our PhD studies but also after. And we still collaborate with LEP e.g. we write at the moment joint papers and even a book (see [B8]) and supervise joint PhD students.

## 6 LEP as the remarkable broad and innovative researcher in mathematical sciences and beyond

LEP has written 16 books and around 320 papers in international Journals. He is also well cited. For example at Google Scholar he has at the moment around 6900 citations, his h-index is 34 and his 10-index is 127. In particular, he has written joint books or papers with more than 150 researchers from 39 different countries. These facts support that he is a remarkable popular, broad and innovative mathematician. In this Section we describe shortly within which areas LEP has mainly given his contributions and also give examples of such contributions. This Section may be regarded not only as an updating of LEP's scientific contributions as described in [A], in fact it also essentially complements this description to give a more complete and broad picture of LEP's total scientific merits. Moreover, some important moments and persons which, in our opinion, in a crucial way have influenced LEP in this development and his choice of problems are also mentioned below. LEP has been a supervisor for students with PhD exams in each of the eight areas described below.

### 6.1 Fourier analysis

This was the subject of LEP's PhD thesis [1]. His first publication from 1973 was a joint paper [2] with his supervisor professor Ingemar Wik, who, in its turn, was student to the famous Swedish professor Lennart Carleson. Hence, LEP is a scientific grandson of Lennart Carleson. After that LEP wrote an own paper [3], where he developed ideas which later on was useful also to contribute to other areas described below. By considering LEP's list of publications and the themes of some PhD students of LEP it is clear that now and then LEP has worked partly within this area and this still continues (see e.g. the paper [33] from 2019).

One fairly remarkable innovative research in this area was developed together with Professor Nicolae Popa, Bucharest, Romania and some of their joint PhD students. In particular, they discovered the fact that diagonals in matrices could be regarded as substitutes for Fourier coefficients and that in this way it was possible to prove many classical results in this different frame. In fact, they initiated what is nowadays called Matriceal Harmonic Analysis. The known results up to 2014 were presented and applied in their book [B5].

Another more recent collaboration of LEP is with Dr George Tephnadze, which took his PhD only 25 years old with LEP as a supervisor (professor Ushangi Goginava was also supervisor). They worked with Fourier analysis on some locally compact Abelian groups and proved some strong convergence results and boundedness of some summability methods of Vilenkin-Fourier series in the martingale Hardy spaces. In the frame of this research they have also a successful and fruitful collaboration with Hungarian mathematicians. In particular, they have agreed to present their results in a joint new book manuscript [B9]. This is another proof of the fact that LEP is still very active and innovative also in this his original area.

### 6.2 Interpolation theory

In 1982 LEP was invited to deliver a lecture at a meeting of SMS (Swedish Mathematical Society). The famous Swedish professor Jaak Peetre was the president of SMS at this occasion. Professor Peetre informed LEP that if he could transfer his ideas from this lecture (based on [4]) he had a good chance to prove a difficult open problem in interpolation theory. This was the starting point of LEPs interest in this area. And, in fact, professor Peetre was correct and it led to that LEP could prove his first

result in this area (see [5]), namely a concrete description of Lions-Peetre real interpolation spaces in so called off-diagonal cases. The corresponding concrete formula is sometimes called "the Persson formula", see e.g. [A]. This influence of and collaboration with professor Jaak Peetre has been very important for LEP. During the time he worked with interpolation theory he was several times invited for shorter or longer periods to Lund University and could live for free in his home. And this very tight contact and friendship with Professor Peetre has obviously influenced LEP much more than only in interpolation theory, which can be seen also in the descriptions of LEP's contributions to some of the areas below and even in his positive way to behave e.g. visavi PhD students.

Several of LEP's contributions in interpolation theory, especially those related to his collaboration with professor Lech Maligranda, have been described in a good way in [A] so concerning this part we just refer to the descriptions there. Let us just again pronounce LEP's own important and well cited paper [6] concerning his method "interpolation with a parameter function". Moreover, also for example LEP's paper [12], jointly written with professor Maria Jesus Carro and their joint PhD student, concerning s-divisibility and Holmstedt's formula should be mentioned. Finally, we mean that the paper [10] should be mentioned both here and in 6.7.

However, maybe the following is the most remarkable complement to what was presented in A] because it also gives another example of LEP's creative and innovative way of thinking in new situations: The classical interpolation theory, both concerning real, complex, or more general methods, usually concerns interpolation between two Banach (or quasi-Banach) spaces but also some attempts to interpolate between three or more Banach spaces are known (see e.g. [16]). During a period LEP, together with the professor Ljudmila Nikolova, was working on the problem to developing a theory concerning real interpolation between infinite many Banach spaces, even so called scales of such spaces. But during the time they began to publish some preliminary results on this topic it turned out that also professor Maria Jesus Carro from Barcelona had begun to publish results in this direction. In this situation LEP invited these two researchers to his home university to try to together with them create a unified and more final solution of this challenging problem. And both accepted the invitations and after a time of very creative and innovative work and, by also involving the master himself professor Jaak Peetre, they really succeeded to develop a fairly satisfying theory concerning real interpolation between scales of Banach spaces, see [12].

### 6.3 Homogenization theory

In [A] the description of LEP's contributions in this area is only focused on the papers [14] and [15] with e.g. professor Jaques-Louis Lions as a co-author. This is of course one important information but here we want to complement this picture in some important ways, where the most essential one is the following: LEP was the one who in around 1988 initiated this area in Sweden together with his first PhD student Nils Svanstedt. One important step here was that LEP took contact with one of the leading experts in this area professor Gianni Dal Maso, Trieste, Italy, to help him in this initial stage. After that he applied for and received grants for Nils to stay one year and work with professor Dal Maso. After this LEP worked intensively in his typical innovative way to build up this area as an independent discipline in Sweden and later on also in Norway and the final outcome was surprisingly good. In particular, more than 10 students have finalized their PhD exams in this area and with LEP as a supervisor. Moreover, six of these students became later on professors at four different Swedish universities (Nils Svanstedt, Anders Holmbom, Peter Wall, and Niklas Wellander) and one Norwegien university (Dag Lukkassen and Annette Meidell).

LEP's original motivation to build up this area was partly fairly application oriented, which also can be seen from several of his early papers, see e.g. [9]. And this fact was very important during the time LEP built up the area since the obtained results were also important in some engineering sciences and in some cases even for industry so he could get good economic support for employing PhD students etc. All these six mentioned professors have nowadays themselves been responsible supervisors for students with PhD exams in homogenization theory.

Another important contribution of LEP was that he together with his students and professor John Wyller wrote the booklet [B1], which has been surprisingly much cited also outside the mathematical sciences since it is partly also engineering oriented. Finally, we also mention the paper [18] concerning some iterated means in homogenization theory, where also professor Jaak Peetre was involved.

### 6.4 Hardy-type inequalities

In 1990 LEP was invited speaker at a spring school in Roudnice Nad Ladem, Czeck Republic. In his lecture LEP presented his general ideas how to prove inequalities e.g. via convexity theory. One in the audience was professor Alois Kufner, and he directly showed great interest. In particular, he asked if LEP thought it was possible to handle also Hardy-type inequalities in this way. This was the starting point that LEP began to study these inequalities e.g. in Kufner's book on the subject. And this led to a very important and fruitful research collaboration and friendship between LEP and professor Kufner.

The "couple" Kufner/Persson have written many joint papers but without comparison their most important work is the highly cited book [B6], where, in particular, many of the results of LEP's papers in this area are included and applied. Professor Natasha Samko is the co-author of this book. Kufner/Persson also, together with Maligranda, have written another fairly well cited book [B3] concerning the Hardy inequality and its history from the day G.H. Hardy proved his famous inequality in 1925 till 2007. In this conection we also mention the book [B4], jointly written with professors Vakhtang Kokilashvili and Alexander Meskhi. This book concerns more general integral inequalities but even when restricting to the case with Hardy-type inequalities it complements and sometimes even improves the results in [B3] and [B6]

Some of LEPs contributions in Hardy-type inequalities up to 2009, were described in [A], mostly those which were related to his collaboration with the author, so for this part we just refer to the descriptions given there. Let us just remind about the paper [20], where the fascinating prehistory with 10 years of research until Hardy proved his inequality is described in detail. Moreover, in [A] it was described the so called "Persson-Stepanov" condition to characterize Hardy-type inequalities. Another important equivalence result which should be mentioned in this connection is that presented in [21].

Some of LEPs most innovative and important contributions to this area have been developed and published after the publication of [A]. Most of these results can be found in the book [B6] from 2017. Just as typical such examples we mention LEP's contributions in the papers [24], [27], [28], [29] and [32]. We finalize this part by shortly describing two examples of LEP's creative way of thinking:

1.From the above it is clear that the classical "Muckenhoupt-Bradley condition "(see e.g. [B6]) to characterize some Hardy-type inequalities can equivalently be replaced by the "Persson- Stepanov" condition. But with experience from interpolation theory described above to work with scales of Banach spaces instead of two spaces it was natural to think that it could be possible that also these two conditions could be replaced by infinite many conditions. maybe even scales of conditions. Together

with a good PhD student (Anna Wedestig) LEP really found the first results in this direction. Later on and together with professors Alois Kufner, Amiran Gogitashvili and Vladimir Stepanov it was proved that really the Muckenhoupt-Bradley condition could be replaced by four infinite scales of conditions, the so called Muckenhoupt scale, the Persson-Stepanov scale and their dual scales. But later on the same authors even proved that there exist even more characterizing scales outside these four fundamental scales of conditions to characterize the Hardy inequality. Some similar results can also be done concerning "the Mazya-Rosin condition" and "the Stepanov-Persson condition for the case q ¿ p" to characterize the Hardy inequality for the case q ¿ p. All these results are unified and presented in Chapter 7 of the book [B6].

2. Concerning Kufner's original question if also Hardy type inequalities could be proved by using instead a convexity argument the answer is sometimes yes, at least for the power weighted case and when p = q. The history and new developments of this fact was presented in the paper [28], jointly written with professor Natasha Samko. As can be seen from the title of this paper it could have changed the history if Hardy himself had discovered this simple and natural proof instead of the one he finally presented in the 1925 paper. Moreover, in this paper it is clearly proved that in principle all Hardy weighted inequalities for fixed p = q are equivalent, and this is not clearly pointed out in most analysis books. In a later paper [29], now with also professor Sorina Barza as coauthor, they also considered a limiting case and improved in a fundamental way a previous result of C. Bennett, which he found in his important studies in interpolation theory. This new inequality contains two constants and both constants are sharp. The present authors do not know any other inequality with this remarkable property.

Summing up, we mean that LEP has done remarkable innovative and great contributions in this area. Some of our friends call him "King of Hardy inequalities" or "Hardinjo" (little Hardy), and we judge that he really is worth these "titles". In particular, we know that LEP nowadays has something like 15 different proof of Hardy's original inequality.

### 6.5 Convexity and Inequalities

Concerning convexity LEP's greatest merit so far is the well written and highly cited book [B7], where, in particular, many of the results in his papers in this area are included and applied. The co-author of this book is Professor Nicolae Popa. Also inequalities was an important research area for LEP during all his career. Already during his PhD thesis work he worked partly by improving classical Fourier inequalities and this interest has continued, see 6.1. Moreover, the work and merits of LEP concerning Hardy type inequalities has been reported on in 6.4. However, here we will focus on another fact namely that LEP may be regarded as an expert concerning inequalities far beyond the two examples given above. We mean that especially his main idea that almost all classical inequalities can be proved and improved by using some simple convexity argument e.g. via Jensen's inequality (correctly formulated this inequality is more or less equivalent to the concept of convexity) is especially important. This fact has been described and exemplified e.g. in his Lecture Notes [31] in Pierre-Louis Lions' seminar and in the review paper [30], jointly written with professor Natasha Samko. Moreover, LEP has been an author of several papers related to the crucial Jensen inequality. Let us here just as one example mention the remarkably well cited paper [11] concerning new thoughts on the classical Hadamard's inequality. Another important contribution by LEP in this area is the book [2], where, in particular, many of LEP's results in his papers concerning Carlson type inequalities, and their close relation to interpolation theory, are included and applied. The co-authors here are his former student Dr Leo Larsson and professors Lech Maligranda and Josip Pecaric. As a final example we mention the

early paper [7] concerning improvements of Bechenbach's inequality with applications. This paper was jointly written with professor Jaak Peetre.

### 6.6 Function spaces and functional analysis

Several of LEPs contributions concerning function spaces and functional analysis till 2009, have been described in [A], especially those related to his collaboration with the author, so concerning this part we just refer to the descriptions given there. However, we will here complement and update this information.

First of all we refer to LEP's own early paper [4], where he gave an alternative exact description of classical Lorentz spaces on a form that it can be used to obtain embedding results of Lorentz spaces into other function spaces. This description is of the same type as the previously mentioned "the Persson formula" (see also [A]). Another important paper is [17], where well known Jacques-Louis Lions' formula for reproducing kernels of harmonic and other functions was investigated and applied in a very genius way. Moreover, we refer to [26], where, due to the authors' expertise in working with Hardy type inequalities (see 6.4), it turned out that there was possible to study mapping properties of potential-type operators in such non-standard spaces as weighted generalized Morrey spaces.

Finally, we mention that Hardy-type inequalities usually are studied in the frame of weighted Lebesgue spaces. However, in Chapter 7 of the recent book [B6] it is included a section, where Hardy-type inequalities are also studied in other functions spaces. In particular, the contributions in recent papers by the authors of the book (e.g. concerning weighted generalized Morrey and weighted Hlder spaces) are included and discussed in this more general frame:

### 6.7 Applied Mathematics

In a great part of his most innovative research LEP has obviously had concrete applications in mind when he chosen his problems to investigate. In particular, when he introduced and developed homogenization theory as a new research area in Sweden and Norway (see 6.3) these applications were taken from material science and composites engineering sciences, see e.g. [9]. Moreover, many of his works in Fourier analysis (see 7.1) and the problems handled in the PhD theses of his students in this area have obviously been motivated by concrete applications e.g. in image analysis, signal processing or problems related to beam bending (bridges, tunnels, dams).

In [8], LEP together with Professor Timo Koski, proved some results concerning generalized entropies with applications to compression of data. In [10] the same authors, together with professor Jaak Peetre , used their new ideas concerning entropies combined with interpolation theory (see 6.2) to point out a new application concerning linear communication channels. Moreover, in [23] LEP, together with professor Ryskul Oinarov and their joint student, applied and developed their knowledge from Hardy type inequalities (see 6.4) to point out some spectral properties of a class of important differential operators.

In [25] ideas from homogenization theory and nonlinear variational methods were used to derive effective properties of multiscale materials.

Maybe the mostly known contribution of LEP in this area is that he has developed a very popular course for PhD students in other subjects (than mathematics). The course consists of an introduction to 10 different applied areas of special interest in engineering sciences. Each part is usually introduced

via an applied example from the corresponding area. The course is web-based and easily available from LEP's homepage. LEP has given the course several times at UiT The Artic University of Norway, Luleiversity of Technology and Uppsala University. Once it was even around 50 PhD students from around 20 different disciplines taking his course. This material has recently been rewritten and complemented in a form so it fits to the style of the new book [B8].

### 6.8 Historical and didactic contributions

Some of LEP's plenary talks at international conferences have been given as specially designed historical related talks about the researchers to whom the conference were dedicated to. Abbreviations of some of these talks have been published in international Journals or books, see e.g. those dedicated to Professors Jaak Peetre (see [B]), Gunnar Sparr (see [C]), Rykul Oinarov (see [D] and Josip Pecaric (see [E]).

LEP has also been very interested in teaching, In fact, he is maybe the only Swedish professor in mathematics, which has diploma to be teacher also on levels under university level. He has got a special award as best university teacher of the year (all subjects) . Another year he got the award as best supervisor of the year (all subjects) at his university. He has also been co-author of some papers in very good international Journals in this direction. For example in [19], jointly written with e.g. the well-known mathematics education professor Barbro Grevholm, his model for PhD education was investigated and discussed. Moreover, in [22], jointly written with e.g. the well-known mathematics education professor Anna Sierpinska, the PC recorded video lectures developed by us at The Arctic University of Norway, were presented and investigated in a scientific way.

The most prestigious award in this direction LEP has got is "Ångpannefreningens" award (100 000 SEK appr. 10000 Euro) for his outstanding work to transfer knowledge to the world outside the university. This award is given each year for at most one researcher from all subjects and all universities in Sweden.

## 7 Final words in this paper about Lars-Erik

LEP has been married two times for totally 52 years. He has five children and five grandchildren. LEP loves his family and a more detailed description of "LEP's two families" can be found as an own section of [A].

LEP is also very well-known outside the university community e.g. depending on his popular mathematically innovative lectures at basic schools and television in Sweden together with many articles about him in newspapers.

LEP is also well known for a broad community concerning the fact that he has participated, and always finalized, the famous Swedish ski competition "Vasaloppet", 90 km skiing, at 70 different occasions (25 of them as so called "open track"). At eight different occasions he has even succeeded to do three "Vasalopp" the same year. His best time in Vasaloppet is 4.56. Moreover, he has participated, and always finalized, 19 Maraton runnings (42 km) with the best time 2.49. More information of similar type can be found in an own Section of "LEP in competitions" in [A].

Summing up what we have written above we hope that we have clearly motivated why we have chosen this title of our article so we finalize by pronouncing:

**Thank you Lars-Erik for everything, in particular,**

* for all wonderful new knowledge and innovative ideas you have given to the mathematical sciences.

* for providing us collaborators, including all PhD students, with your positive and supportive spirit and ideas to mathematics and life.

* for always being open for and looking forward new adventures in the future.

* for being our Pers(s)onal friend and still a very active main collaborator and ideal for us.

**Lars-Erik: We hereby give you our warmest and most cordial congratulations to your +25 birthday and hope for many years more in collaboration and life with "the happy mathematician".**

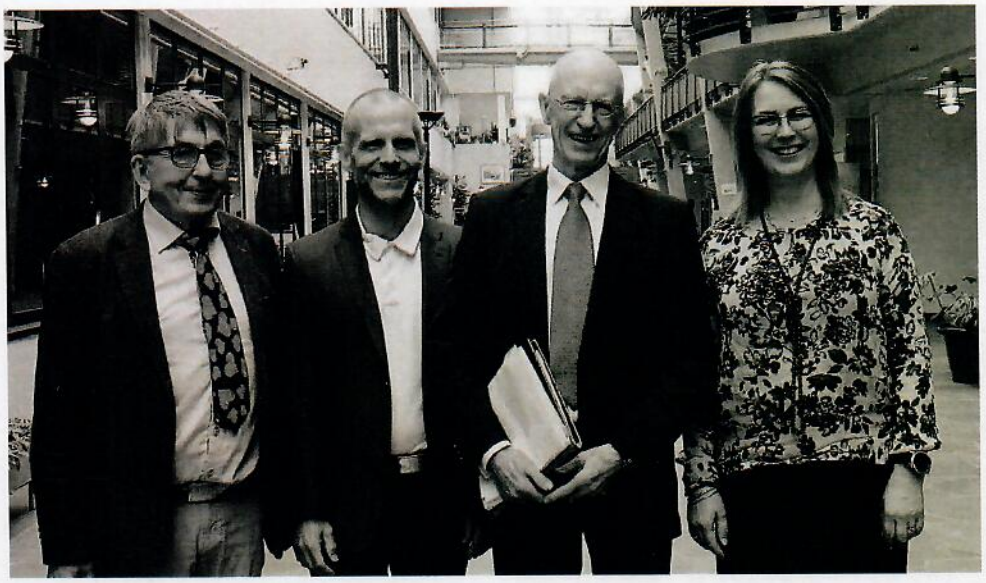

**fig.3:** In the photo you see LEP at UiT in Narvik together with us and our latest PhD Ralph Høibakk. His creative thoughts of mathematics and life have many times been discussed and initiated at this institution, where he has worked since 1992.

## 8 Publications

**Selected books (out of 16)**

[B1] L.E. Persson et al. , The homogenization method - An introduction, Studentlitteratur Publ., Lund, 1993 (86 pages).

[B2] L. Larssson, L. Maligranda, J. Pecaric and L.E. Persson, Multiplicative Inequalities of Carlson Type, World Scientific Publ., 2006 (201 pages).

[B3] A. Kufner, L. Maligranda and L.E. Persson, The Hardy Inequality. About its History and Some Related Results, Vydavatelsky Servis Publ., 2007 (161 pages).

[B4] A. Meskhi, V. Kokilashvili and L.E. Persson, Weighted Norm Inequalities and Schur Multipliers, Nova Scientific Publ., Inc., 2009 (329 pages).

[B5] N. Popa and L.E. Persson, Matrix Spaces and Schur Multipliers. Matriceal Harmonic Analysis, Wold Scientific Publ., 2014 (208 pages).

[B6] A. Kufner, L.E. Persson and N. Samko, Weighted Inequalities of Hardy Type, second Edition, World Scientific Publ., 2017 (480 pages). The first Edition was published in 2003.

[B7] C. Niculescu and L.E. Persson, Convex Functions and Their Applications. A Contemporary Approach, CMS Books of Mathematics, Springer, 2018 (431 pages). The first Edition was published in 2006.

[B8] D. Lukkassen, A. Meidell and L.E. Persson, Modelling of Engineering Problems in Applied Mathematics, book manuscript to be submitted (215 pages)
[B9] L.E. Persson, G. Tephnadze and G. Weisz, Martingale Hardy Spaces and Summability of the Onedimensional Vilinkin-Fourier Series. book manuscript to be submitted (around 350 pages).

**Selected further publications (out of around 300)**

[1] L,E. Persson, Relations between regularity of periodic functions and their Fourier series, PhD thesis, Umeiversity, 1974.
[2] L.E. Persson and I. Wik, Integrability conditions on periodic functions related to their Fourier transforms, J. Math. Inequal. Appl. 44 (1973), 291-309.
[3] L.E. Persson, Relations between summability of functions and their Fourier series. Acta Math. Acad. Sci. Hungar 27 (1976), No. 3-4, 1976, 267-280.
[4] L.E. Persson, An exact description of Lorentz spaces, Acta Sci. Math. 46 (1983), 106-116.
[5] L.E. Persson, Descriptions of some interpolation spaces in off-diagonal cases, Lecture Notes in Mathematics 1070 (1984), 213-214.
[6] L.E. Persson, Interpolation with a parameter function, Math. Scand. 59 (1986), No. 2. 199-222.
[7] J. Peetre and L.E. Persson. General Beckenbach's inequality with applications, In: Function Spaces, Differential Operators and Nonlinear Analysis, Pitman Research Notes in Mathematics 211 (1989), 125-139.
[8] T. Koski and L.E. Persson, Some properties of generalized entropies with applications to compression of data, Journal of Information Sciences 62 (1992), 103-132.
[9] A. Holmbom, L.E. Persson and N. Svanstedt, A homogenization procedure for computing effective moduli and microstresses in composite materials, J. Composite Engineering 1 (1992), No. 1249-1259.
[10] T. Koski, J. Peetre and L.E. Persson, e-entropy, e-rate and interpolation spaces revisted with an application to linear communications channels, J. Math. Anal. Appl. 186 (1994), 265-276.
[11] S.S. Dragomir. J.E. Pecaric and L.E. Persson, Some inequalities of Hadamard type, Soochow J. Math. 21 (1995), No. 3, 335-341.
[12] M.J. Carro, L. Nikolova, J. Peetre and L.E. Persson, Some real interpolation methods for families of Banach spaces, J. Approx. Theory 89 (1997), 26-57.
[13] M.J. Carro, S. Ericsson and L.E. Persson, S-divisibility properties and a Holmstedt type formula, J. Approx. Theory 98 (1999), 1-24.
[14] J.L. Lions, D. Lukkassen, L.E. Persson and P. Wall, Reiterated homogenization of monotone operators, C.R. Acad., t.330, Ser.I (2000), 675-680.
[15] J.L. Lions, D. Lukkassen, L.E. Persson and P. Wall, Reiterated homogenization of nonlinear monotone operators, Chin. Ann. Math. 22B.1(2001), 1-12.
[16] I. Asekritova, N. Krugljak and L.E. Persson, Lions-Peetre reiteration formulas for triples and their applications, Studia Math. 145 (2001), 219-253.
[17] M. Englis, D. Lukkassen, J. Peetre and L.E. Persson, On the formula of Jacques-Louis Lions for reproducing kernels of harmonic and other functions, J. Reine Angew. Math. 570 (2004), 89-129.
[18] D. Lukkassen, J. Peetre and L.E. Persson, On some iterated means arising in homogenisation theory, Applications of Math, 49 (2004), No. 4, 343-356.
[19] B. Grevholm, L.E. Persson and P. Wall, A dynamic modal for education of doctoral students and guidance of supervisors in research groups, Educational Studies of Mathematics 60 (2005), No. 2, 145-170.
[20] A. Kufner, L. Maligranda and L.E. Persson, The prehistory of the Hardy inequality, Amer. Math. Monthly 113 (2006), No. 8, 715-732.
[21] C. Okpoti, L.E. Persson and and G. Sinnamon, An equivalence theorem for some integral conditions with general measures related to Hardy's inequality, J. Math. Anal. Appl. 326 (2007), No. 1, 398-413.
[22] D. Lukkassen, L.E. Persson and A. Sierpinska, Some aspects on web-courses in mathematics on PC screen recorded video lectures, NOMAD (2007), No, 4, 1-11.
[23] A. Kalybay, R. Oinarov and L.E. Persson, Spectral properties of a class of differential operators, Math. Inequal. Appl. 1 (2007), No. 3, 355-376.
[24] A. Gogatishvili, A. Kufner and L.E. Persson, Some new scales of conditions of the class Bp, Acta Math. Hungar. 123(2009), No. 4, 365-377.

[25] D. Lukkassen, A. Meidell and L.E. Persson, Nonlinear Variational Methods for Estimating Effective Properties of Multiscale Materials, In: Nonlinear Analysis and Variational Problems, 367-414, Optimization and its Applications, Springer, New York, 2010 (Online 14 September 2009).
[26] L.E. Persson and N. Samko, Weighted Hardy and potential operators in the generalized Morrey spaces, J. Math. Anal. Appl. 377 (2011), no. 2, 792-806.
[27] L.E. Persson and N. Samko, What should have happened if Hardy had discovered this ? J. Inequal. Appl. 2012:29 (20 pages).
[28] S. Barza, L.E. Persson and N. Samko. Some new sharp Hardy-type inequalities via convexity. J. Inequal. Appl. 2014:6 (10 pages).
[29] O. Popova, L.E. Persson and V. Stepanov, Weighted Hardy-type inequalities on the cone of quasi-concave functions, Math. Inequal. Appl. 17 (2014), No. 3, 879-898.
[30] L.E. Persson and N. Samko, Inequalities and Convexity. In: Operator Theory, Operator Algebras and Applications, 279-306, Oper. Theory Adv. Appl. 212, Birkher/Springer, Basel, 2014.
[31] L.E. Persson, Lecture Notes, P.L. Seminar (November 2015) (77 pages).
[32] R. Oinarov, L.E. Persson and S. Shaimardan, Hardy-type inequalities in fractional h-discrete calculus, J. Inequal. Appl. 2018: 73 (14 pages).
[33] G. Akishev, L.E. Persson and A. Seger, Some Fourier inequalities for orthogonal systems in Lorentz-Zygmund spaces, J. Inequal. Appl. (JIA), (2019), 2019:171 (18 pages).

**Further publications with Pers(s)onal focus**

[A] L. Maligranda, Lars-Erik Persson-the man and his work. Banach J. Math. Anal. 4(2010), No. 1, 185-201.
[B] M. Cwikel, R. Rochberg , L.E. Persson and G. Sparr, Jaak Peetre: the man and his work. In: Function Spaces, Interpolation Theory and related Topics, W. de Greuter (Eds: M. Cwikel et al.), Berlin, 2000, 1-37.
[C] L.E. Persson, Gunnar Sparr: The remarkable mathematician, entrepreneur and ambassador for mathematics, Analysis for Science, Engineering and Beyond, The Tribute Workshop in Honour of Gunnar Sparr held in May 8-9. 2008 (Eds: K. Åström and L.E. Persson), Springer, 2012,1-22 (online 18 May 2011).
[D] L.E. Persson, The way to the top of science of professor R. Oinarov, Eurasian Math. J. 8(2017), No. 2, 9-21.
[E] A.A. Aljinovic, I. Peric and L.E. Persson, Josip Pecaric-and his life in mathematics and politics, Math. Inequal. Appl. , to appear in No. 4, 2019.